# Performance Evaluation of Medical Imaging Service

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## ABSTRACT

Public/private decision makers have faced challenges to improve healthcare services for supporting the increasing demand and simultaneously reducing the associated costs. Although the adoption of information and communication technologies (ICTs) are important in this context, the current service status should be firstly examined and different configurations/scenarios quantitatively evaluated before any further adjustment. Formal methods are of great importance, since they provide mathematical means for quantitative evaluation of systems and allow property analysis/verification. This paper presents an approach based on stochastic Petri nets for the performance evaluation of medical imaging service, adopting a real-world case study to demonstrate the feasibility of the proposed approach.

## Categories and Subject Descriptors

H.4 [**Information Systems Applications**]: Miscellaneous

## General Terms

Performance, Management

## Keywords

Performance Evaluation, Formal Methods, Business Process Management

## 1. INTRODUCTION

Over the years, healthcare institutions have dealt with greater pressures for increasing service capacity and quality as well as, at the same time, reducing related costs. Moreover, new procedures and techniques are always being incorporated, such as those based on medical imaging, which intensify the challenges for the decision makers to meet previous requirements [1].

Indeed, medical imaging is of utter important in healthcare, since the related techniques and procedures considerably improves, for instance, the diagnostic of diseases, usually in a non-intrusive manner. Information and communication technologies (ICTs) have provided important contributions in this field, allowing images to be digitally generated, and, so, providing better mechanisms for transmission and storage. ICTs may also improve the related business processes in medical imaging services, as activities may be simplified or even extinguished. However, a proper planning should be performed previously, such that the current situation is evaluated and the benefits of ICTs are estimated before any further adjustment.

Quantitative data (i.e., performance metrics) provide important insights to the decision maker, and lay down a basis for further adjustment in the process. Without loss of generality, this is one of the business process management (BPM) goals [2], in the sense that the current process is evaluated and improvements are proposed. Formal models are very useful in this context, since they adopt mathematical means for representing systems and allow property analysis/verification. Petri nets (PN) [3] are an appropriate family of formalisms very well suited for modeling systems exhibiting concurrency, synchronization and communication mechanisms. Besides, a prominent stochastic extension, more specifically, stochastic Petri nets (SPN) [4], also allows representing the timing behaviour of activities using probability distribution functions (i.e., exponential distribution) and such an extension has been adopted to evaluate the performance of several systems.

This paper presents an approach for evaluating the performance of medical imaging service, using a computed tomography service of a public teaching hospital as a real-world case study. The approach relies on stochastic Petri nets for modeling the service and, consequently, for estimating further adjustments, which include the deployment of a picture archiving and communications system (PACS) [5].

The rest of the paper is organized as follows. Section 2 summarizes related works and Section 3 provides an overview of medical imaging services. Section 4 presents the adopted methodology and Section 5 describes the computational model (i.e., SPN) . Section 6 presents a real-world case study and Section 7 concludes this work.



## 2. RELATED WORKS

The stochastic modeling of medical/healthcare services has received considerable attention over the years in order to provide means for a better assessment of the current service and the possibility to estimate further adjustments.

In [6], the authors present a technique based on coloured

Petri nets for redesigning a business process at a mental healthcare institute. Barjis [7] proposes a business process modeling method (inspired on Petri nets) and demonstrates the respective feasibility in a patient admission process. Dotoli et al. [8] propose a timed Petri net model that describes the pulmonology department of a hospital. In that work, the PN model provides means for evaluating different inventory management policies.

Jiang el al. [9] present a queueing model to evaluate an urgent care center with the purpose of examining the possibility of incorporating parallel activities for shortening the patient cycle time. In [10], Cochran and Roche present a multi-class queueing network for evaluating the performance of a hospital emergency department. In [11], the authors describe an analytical model based on queueing network for capacity planning of neonatal intensive care unit.

Different from previous works, this paper presents an approach based on stochastic Petri nets for modeling medical imaging service, adopting a computed tomography service as a case study. The approach evaluates the current status as well as it provides mechanisms for examining further adjustments in the service.

## 3. MEDICAL IMAGING SERVICES

Over the years, medical imaging has considerably improved the ability of physicians to diagnose several diseases in patients and it has also enhanced surgical procedures due to image-guided interventions. With the advent of computer-related technology, digital imaging has positively impacted medical and healthcare services, including those based on telemedicine, due to the inherent suitability for transmission and storage. Computed tomography, magnetic resonance imaging and ultrasonography are representative modalities that adopt digital imaging in some manner.

Recently, debates have occurred about the risen costs concerning medical imaging services and the actual benefits for the patients [12]. Indeed, medical images are important in several contexts, since they provide means for better diagnostics, minimally-invasive procedures, and, thus, shorter hospitalization, possibly reducing healthcare costs. Nevertheless, due to those debates, decision makers are facing challenges to reduce service costs and, concurrently, to support the increasing demand. Modifications in the service may improve the response time, the costs or even both, and stochastic models play a fundamental role in estimating performance metrics before changing the service. Decision makers would then possess an important tool to assess the best arrangement for the desired service. In general, stochastic modeling provides mechanisms for representing activities that do not possess a deterministic behavior (due to intrinsic variations in their executions). In such a case, the activities are represented by probability distributions.

As an example, consider the activity diagram depicted in Figure 1, which represents a computed tomography service of a real, public teaching hospital. After the proper authorization, a patient requests an appointment, and, on the scheduled day, the image acquisition is performed. Although the acquired images are digital, the hospital does not have an infrastructure for a PACS. Thus, after some period, the images are printed, and a resident physician makes the report. Finally, a typist produces the final document based on the physician's manuscript, and such document is made available to the user. Although the process has been described taking into account the main flow, other alternative flows are assumed, such as if the acquired images do not possess a good quality, another image acquisition may be required. As the reader should note, ICTs could significantly improve the process by eliminating some intermediary activities.

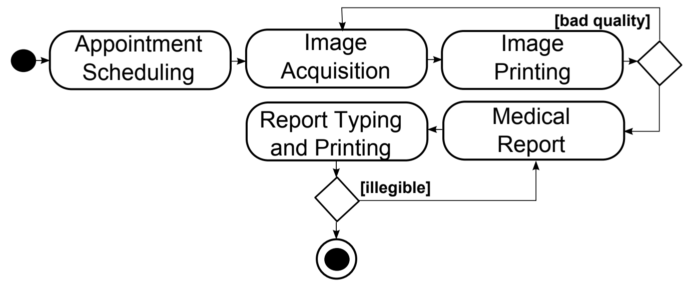


**Figure 1: Activity Diagram of a Computed Tomography Service**

## 4. METHODOLOGY

This section presents the adopted methodology for evaluating medical imaging service. Formal models are important to estimate performance metrics related to different configurations in a medical service, however, decision makers may not have the required knowledge to utilize those models. Thus, higher level models, such as Business Process Modeling Notation (BPMN) and Unified Modeling Language (UML) [13], may be initially adopted to represent the current status, and, next, specialists may create the respective formal representation to evaluate different arrangements. As follows, the methodology steps are presented taking into account a sequential execution, but an iterative mechanism can also be considered:

- *Problem Understanding.* First of all, the current service must be understood to better visualize the activities, the respective relations, and possible points for improvement. In this step, UML and BPMN models are feasible representations, since they provide an interesting overview of the activities without formal details;
- *Metric Definition.* Once the service is well comprehended, performance evaluation specialists (in conjunction with imaging service managers) define the metrics, which are taken into account to examine the current status and to lay down a basis for further comparison with other configurations;
- *Measuring.* Next, the measuring activity starts, which contemplates gathering enough data to estimate the metrics previously defined;
- *Formal Modeling.* The high-level model is mapped into a formal representation, more specifically, into stochastic Petri nets, by combining basic building blocks using formal composition rules. Besides, the collected data in previous step are adopted to refine the SPN model with timing information and to select the probability distribution function for each modeled activity;
- *Validation.* The formal model is then validated considering the current status. Some metrics are considered

in this step, and hypothesis tests indicate the suitability of the SPN model to represent the service;

- *Scenario Definition and Evaluation.* Assuming the model validation, the team may conceive new scenarios/arrangements from the baseline model and, next, they may estimate the respective metrics from analysis or simulation techniques;
- *Service Adjustment.* After evaluating different scenarios, the decision maker has more confidence to execute the adjustments, since the metrics of interest have been estimated, providing important perception concerning the impacts of such adjustments.

Concerning the formal modeling, only the building blocks utilized in the case study are presented due to space restriction.

# 5. STOCHASTIC PETRI NETS

Petri nets (PN) are a family of formalisms very well suited for modeling several types of systems, since concurrency, synchronization, communication mechanisms as well as deterministic and probabilistic delays are naturally represented. In general, a Petri net is a bipartite directed graph, in which places (represented by circles) denote local states and transitions (depicted as rectangles) represent actions. Arcs (directed edges) connect places to transitions and vice-versa. Tokens (small filled circles) may reside in places, which denote the state (i.e., marking) of a PN. An inhibitor arc is a special arc type that depicts a small white circle in one edge, instead of an arrow, and they usually represent the unavailability of tokens in places.

Stochastic Petri nets (SPN) [4] is a prominent PN extension, which allows the association of probabilistic delays to transitions using the exponential distribution, and the respective state space is isomorphic to continuous time Markov chains (CTMC) [14]. Besides, SPNs allow the adoption of simulation techniques for estimating performance metrics, as an alternative to the Markov chain generation. Figure 2 depicts a SPN model and the respective CTMC.

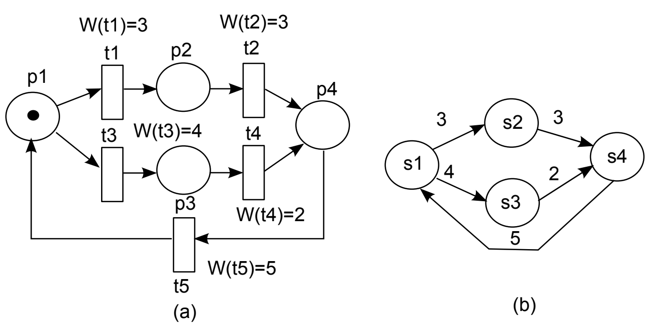


**Figure 2: SPN model (a) and the respective CTMC (b)**

This work adopts generalized stochastic Petri nets (GSPN), which are a renowned stochastic extension based on SPN, assuming timed transitions as well as immediate transitions (i.e., transitions that fire in zero time). Usually, immediate transitions are represented by thin, black rectangles, whereas timed transitions are depicted as thick, white rectangles (see Figure 2). GSPNs have been successfully adopted in evaluating the performance and dependability of several systems (e.g., [15, 16]). Throughout this work, the terms GSPN and SPN are interchangeably adopted . For a better understanding, next paragraphs present the GSPN definition, which is based on [4, 16].

(**Generalized Stochastic Petri Net**) A generalized stochastic Petri net (GSPN) is a bipartite direct graph represented by a tuple $GSPN = (P, T, \pi, I, O, H, \mu_0, W)$, in which:

- $P$ is the set of places;
- $T = T_{imm} \cup T_{timed}$ is the set of immediate ($T_{imm}$) and timed ($T_{timed}$) transitions, such that $T \cap P = \emptyset$;
- $\pi : T \to \mathbb{N}$ is the priority function, such that
$$\pi(t) = \begin{cases} \geq 1, & if(t \in T_{imm}) \\ 0, & if(t \in T_{timed}) \end{cases}$$
- $I, O, H : T \to Bag(P)$ are the input, output and inhibition functions, respectively, in which $Bag(P)$ is the multiset on $P$ ($Bag(P) : P \to \mathbb{N}$);
- $M_0 : P \to \mathbb{N}$ is the initial marking function;
- $W : T \to \mathbb{R}$ is the weight function, which maps a immediate transition into a weight and a timed transition into the respective rate $\lambda_t$:
$$W(t) = \begin{cases} w_t \geq 0, & if(t \in T_{imm}) \\ \lambda_t > 0, & if(t \in T_{timed}) \end{cases}$$

The behaviour of a GSPN is defined in terms of a *token game* (i.e., the evolution of the GSPN marking), in the sense that tokens are created and destroyed according to the transition firings. A transition $t$ is enabled in a marking $M_i$, iff: (i) $\forall p \in P, M_i(p) \geq I(t,p) \wedge I(t,p) > 0$; (ii) $\forall p \in P, M_i(p) < H(t,p) \wedge H(t,p) > 0$; and (iii) $\pi(t)$ is the greatest value among other transitions that (i) and (ii) are met. An enabled transition in marking $M_i$ may fire, leading to a marking $M_j$: $\forall p \in P, M_j(p) = M_i(p) + O(t,p) - I(t,p)$. From the initial marking $M_0$, the state space of the GSPN (i.e., all reachable markings) may be computed, and, next, the CTMC can be obtained.

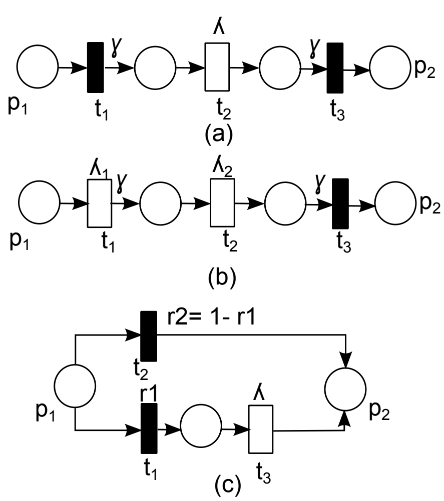


**Figure 3: Phase Approximation: (a) Erlang; (b) Hypoexponential; and (c) Hyperexponential**

Although GSPNs assume the exponential distribution for timed transitions, non-exponential activities can be represented using phase approximation technique. Basically, different combinations of places, immediate and timed transitions are adopted to represent different delay distributions. This paper utilizes the technique described in [17], which, from the mean delay ($\mu_d$) and standard deviation ($\sigma_d$), a phase approximation is taken into account. The following algorithm is considered:

- If $\mu_d = \sigma_d$, only a single timed transition is adopted;

- Assuming $\mu_d/\sigma_d \in \mathbb{N}$ and $\mu_d/\sigma_d \neq 1$, the phase approximation considers an Erlang subnet (Figure 3 (a)), such that $\gamma = (\frac{\mu_d}{\sigma_d})^2$ and $\lambda = \gamma/\mu_d$;

- Considering that $\mu_d > \sigma_d$, a hypoexponential subnet is adopted (Figure 3 (b)) and

$$(\frac{\mu_d}{\sigma_d})^2 - 1 \leq \gamma < (\frac{\mu_d}{\sigma_d})^2 \quad (1)$$

$$\lambda_1 = \frac{1}{\mu_1} \ and\ \lambda_2 = \frac{1}{\mu_2} \quad (2)$$

$$\mu_1 = \frac{\mu_d \pm \sqrt{\gamma(\gamma+1)\sigma_d{}^2 - \gamma\mu_d{}^2}}{\gamma+1} \quad (3)$$

$$\mu_2 = \frac{\gamma\mu_d \mp \sqrt{\gamma(\gamma+1)\sigma_d{}^2 - \gamma\mu_d{}^2}}{\gamma+1} \quad (4)$$

- If $\mu_d < \sigma_d$, the approximation assumes a hyperexponential subnet (Figure 3 (c)), in which

$$r_1 = \frac{2\mu_d{}^2}{(\mu_d{}^2 + \sigma_d{}^2)} \quad (5)$$

$$r_2 = 1 - r_1 \quad (6)$$

$$\lambda = \frac{2\mu_d}{(\mu_d{}^2 + \sigma_d{}^2)} \quad (7)$$

# 6. CASE STUDY

This work adopts the computed tomography service described in Section 3 to demonstrate the adopted approach for evaluating medical imaging services. Following the methodology, the metrics were defined and measured in the real service. Next, the GSPN model was constructed (Figure 4) based on the combination of two building block types: (i) one representing the patient arrival; and (ii) other block modeling the activity and the respective queue. The trapezoids denote generic transitions, which abstract the subnet that describes a phase approximation (Section 5). Besides, the building blocks are combined using the formal composition rules detailed in [15]. Note that the alternative flows are represented by dashed arcs and transitions. However, they are not considered in the model evaluation, since these flows minimally impact the overall results for this case study.

In such GSPN model, appointment scheduling is implicitly represented by the patient arrival, since we are concerned with the mean waiting time for the image acquisition. The mean waiting time ($wait$) is modeled through Little's law [14], $wait = L/\lambda'$, in which $L = E\{\#pacqq\}$ is the average number of patients waiting in the queue and $\lambda' = P\{\#parr > 0\} \times W(tarr) \times (1 - \{P\#pacqq = q1\})$ is the effective arrival rate. More specifically, $E\{\}$ estimates the mean value for the expression enclosed by the brackets; $P\{\}$ indicates the probability of the specified expression; $\#p$ denotes the number of tokens in place $p$; and $q1$ is the queue size of the image acquisition activity. Besides, $W(tarr)$ is defined in such a way that $wait$ equals the waiting time measured in the real service.

The other activities are represented by a different building block. Adopting the image acquisition activity as an example, the patient may enter the queue ($tarrok$) or may be discarded if the queue is full ($tarrl$). The adoption of acceptance and loss transitions is twofold: (i) in image acquisition activity, if the queue is full, indeed, the patient needs to be redirected to another institution due to limitations of the public teaching hospital; and (ii) the arrival rate ($\lambda_{arrival}$) in some activities is greater than the service rate ($\lambda_{service}$). In the latter case, the waiting components would increase boundless, and such an approach keeps the state space finite (in conjunction with a queue size, e.g., $q1$) as well as it does not affect the service rate. Nevertheless, the effective arrival rate for an activity can be obtained as presented previously.

Once the computed tomography machine is available (i.e., a token in place $machine$), the image acquisition is performed and the images (symbolized by a token) are transferred to the next activity.

GSPN allows to estimate the impact of different configurations in an imaging service. As follows, the model validation is firstly presented, and, next, different scenarios are evaluated.

## 6.1 Validation

Over one month, data were daily collected in the imaging service concerning the execution time of each activity and the waiting time for the image acquisition. The collected data were then divided into five groups, and the respective mean value and standard deviation were calculated. Afterwards, five models based on Figure 4 were generated assuming the best phase approximation for each transition.

Table 1 depicts the values estimated by each model concerning the execution time (in minutes) of the imaging acquisition ($Img.Acq. - E$) and the medical report ($Phy.R. - E$) activities. Such table also presents the respective values measured in the imaging service ($Img.Acq. - M$ and $Phy.R. - M$).

This work conducted the t-paired test [18] to compare the values produced by the models (adopting a stationary evaluation [14] using Mercury/ASTRO tool [19]) and the values measured in the hospital. Assuming a significance level of 5%, the test for the image acquisition activity generates the following confidence interval: [-0.0095895,0.0019433]. As the interval does contain 0, there is no statistical evidence to reject the hypothesis of equivalence between the model and the collected values. Concerning the medical report activity, the confidence interval for the t-paired test is [-32.02483, 1.606058] and, thus, the hypothesis of equivalence can not be refuted.

## 6.2 Scenarios

This section presents some conceived scenarios for improv-

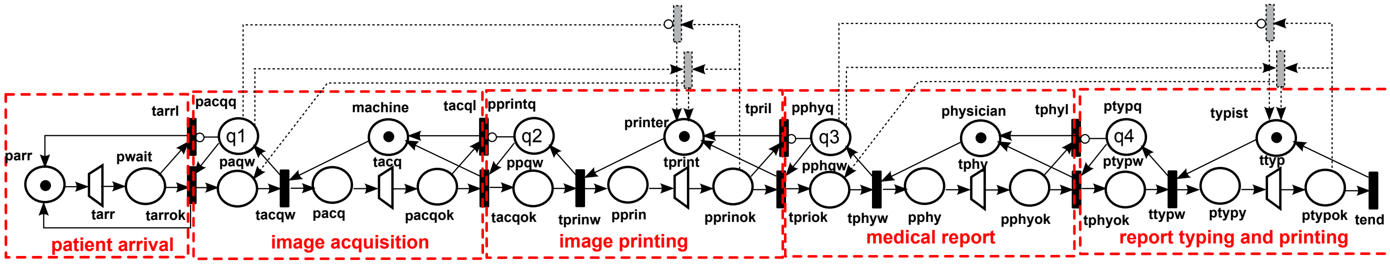


**Figure 4: GSPN model**

**Table 1: Collected data and estimated metrics for the imaging service (in minutes)**

| | Img Acq. - E | Img Acq. - M | Med. R. - E | Med. R. - M |
|---|---|---|---|---|
| 1 | 124.4719 | 124.4727 | 504.8163 | 507.7706 |
| 2 | 103.2797 | 103.2881 | 229.7565 | 245.2304 |
| 3 | 143.9120 | 143.9090 | 117.2682 | 143.9590 |
| 4 | 91.2211 | 91.2359 | 195.7867 | 226.2740 |
| 5 | 114.9684 | 114.9502 | 261.906 | 262.3475 |

ing the computed tomography service. Group 1 is considered as the baseline and Table 2 depicts the values for each timed transition (*trans*) as well as the respective phase approximation (*Phase*). For this Petri net model, one token in place *Physician* actually represents 5 resident physicians, whereas one token in place *Typist* assumes 2 typists. Previous abstraction is related to the structure of the service records for these particular data. Additionally, the reader may argue that the execution times are large, but, due to the characteristics of the public teaching hospital, the physicians and typists are not exclusively allocated for these activities.

**Table 2: Execution times for group 1 (in minutes)**

| Trans | $\mu_d$ | $\sigma_d$ | Phase |
|---|---|---|---|
| tarr | 10.0000 | 10.0000 | Exponential |
| tacq | 124.4719 | 68.6844 | Hypoexponential |
| tprint | 144.93506 | 123.3435 | Hypoexponential |
| tphy | 504.8163 | 455.8848 | Hypoexponential |
| ttyp 4 | 456.0000 | 214.2148 | Hypoexponential |

Initially, we chose to gradually reduce the execution times of each activity, as the utilization factor is more than 90% for all resources. Another resources could be allocated instead, but the cost of a new computed tomography machine is prohibitive and the execution times are large. The adopted metric is the response time of the process, which contemplates the waiting time for the image acquisition until the release of the medical report by a typist. In addition, this experiment assumes a linear cost model, which takes into account the cost of the individuals participating in the process.

The response time for the entire process can be estimated adopting the general response time law [20], $R = \sum_i^n R_i$, such that $R_i$ is the response time of each activity. More specifically, $R_i = wait_i + \bar{x}_i$, in which: (i) $wait_i = L_i/\lambda'_i$, $L_i$ is the average number of components waiting in the queue and $\lambda'_i$ is the effective arrival rate (as previously demonstrated); and (ii) $\bar{x}_i = L_{s_i}/\lambda'_i$ is the mean time performing the activity for $L_{s_i}$ components (e.g., $L_{s_i} = 1 - E\{machine\}$).

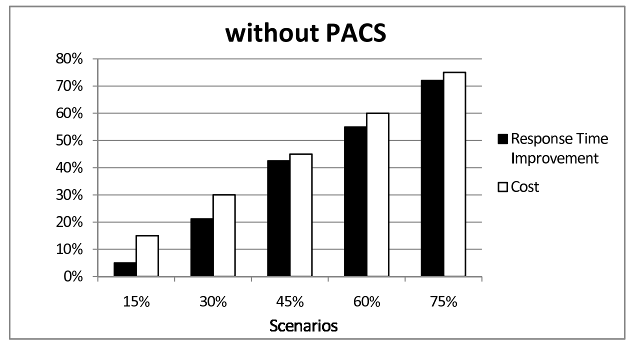


**Figure 5: Scenarios without PACS (in comparison to the baseline)**

Figure 5 depicts the results for some scenarios in comparison to the baseline, considering that the execution times are reduced from 15% to 75% and no PACS is avalaible. These reductions are based on better resource allocations for the activities and training for the personnel. The results demonstrate that the response times improve, but the costs significantly increase. Thus, the decision maker should adopt the scenario that better relates the improvement and the available financial resources for investment.

Figure 6 presents alternative scenarios, in which a PACS is assumed. In this case, the process is simplified, in the sense that some activities (image printing as well as report typing and printing) are no longer required. Disregarding the initial investments for the PACS deployment and taking into account a linear cost model for the personnel, PACS may improve the response time with a significant cost reduction (more than 20%). Only from 45% scenario, the costs may increase. Other benefits may also be obtained, since films and other materials are not required as well as rooms for archiving the physical images. Nevertheless, the maintenance for the PACS incurs costs, but the obtained benefits may considerably surpass this issue.

## 7. CONCLUSION

Formal models and the respective stochastic extensions are of great importance in assessing medical/healthcare services, since they provide an important tool for decision makers to evaluate further modifications in the service. Concerning the debates about the risen costs of medical imaging services, these models play another prominent role, in the sense that different strategies may be evaluated to improve the service before the actual implementation.

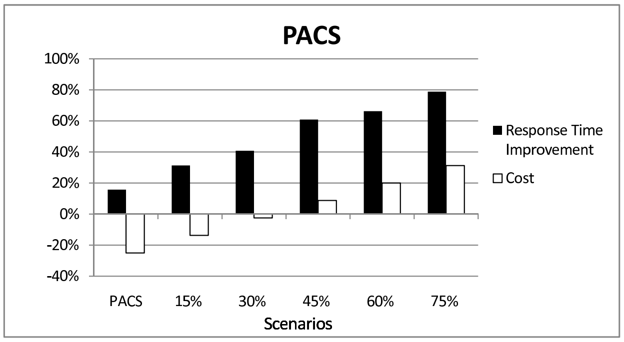


**Figure 6: Scenarios with PACS (in comparison to the baseline)**

This paper presented an approach based on SPN for performance evaluation of medical imaging service, demonstrating the practical application on a real computed tomography service. From basic building blocks, the model was constructed and some scenarios evaluated, including those based on a PACS. As future work, we are planning to define other blocks for estimating metrics related to dependability evaluation.

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